Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA13/01**

Mathematics

International Advanced Level
Pure Mathematics P3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 7 2 1 3 8 A 0 1 3 2

Pearson

1. The functions f and g are defined by

$$f(x) = 9 - x^2 \qquad x \in \mathbb{R} \quad x \geqslant 0$$

$$g(x) = \frac{3}{2x+1} \qquad x \in \mathbb{R} \quad x \geqslant 0$$

(a) Write down the range of f **(1)**

(b) Find the value of fg(1.5) **(2)**

(c) Find g^{-1} **(3)**

1. a) $f(x) = 9 - \underbrace{x^2}$ $\qquad x \in R, \; x \geqslant 0$

x^2 will always be positive for $x \in R$

$\therefore f(x) = 9 - (\text{positive})$

so $\quad f(x) \leq 9$

b) $f(g(1.5))$

$= f\left(\frac{3}{2(1.5)+1}\right)$

$= f\left(\frac{3}{4}\right)$

$= 9 - \left(\frac{3}{4}\right)^2$

$= \frac{135}{16}$

P 7 2 1 3 8 A 0 2 3 2

Question 1 continued

c) to find $g^{-1}(x)$: $g(x) = \dfrac{3}{2x+1}$

① write the function using a "y" and set equal to "x" . $x = \dfrac{3}{2y+1}$

② rearrange to make y the subject : $2xy + x = 3$

$y = \dfrac{3-x}{2x}$

③ replace y with $g^{-1}(x)$.

$g^{-1}(x) = \dfrac{3-x}{2x}$

Because we are told to find $g^{-1}(x)$, we must also state the domain of the inverse function :

↑ domain refers to the set of values we are allowed to plug into our function

domain of inverse function = range of function

↑ range refers to all possible values of a function

∴ domain of $g^{-1}(x)$ = range of $g(x)$

$g(x) = \dfrac{3}{2x+1}$ $x \geq 0$ $x \in \mathbb{R}$

→ when $x = 0$ → $g(x) = 3$

→ as $x \to \infty$ → $g(x) = \dfrac{3/\cancel{x}}{2+\cancel{1/x}}$

$g(x) \to 0$

∴ range of $f(x)$ is $0 < f(x) \leq 3$

equal to 3 when $x = 0$ but never actually reaches 0

↳ only tends to 0

∴ $g^{-1}(x) = \dfrac{3-x}{2x}$

$0 < x \leq 3$

(Total for Question 1 is 6 marks)

2.
$$f(x) = \cos x + 2\sin x$$

(a) Express $f(x)$ in the form $R\cos(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 decimal places.

(3)

$$g(x) = 3 - 7f(2x)$$

(b) Using the answer to part (a),

(i) write down the exact maximum value of $g(x)$,

(ii) find the smallest positive value of x for which this maximum value occurs, giving your answer to 2 decimal places.

(3)

2. a) $f(x) = \cos(x) + 2\sin(x)$

$f(x) = R\cos(x - \alpha)$ ← using compound angle formulae

$f(x) = R(\cos x \cos\alpha + \sin x \sin\alpha)$ $\quad \cos(A-B) = \cos A\cos B + \sin A \sin B$

Compare expanded expression to $f(x)$ given

$R\cos x\cos\alpha + R\sin x\sin\alpha = \cos(x) + 2\sin(x)$

$R\cos\alpha = 1 \qquad R\sin\alpha = 2$

$\frac{\cancel{R}\sin\alpha}{\cancel{R}\cos\alpha} = \tan\alpha = \frac{2}{1}$

$\alpha = 1.107$

$(R\cos\alpha)^2 + (R\sin\alpha)^2$ $\quad \cos^2 A + \sin^2 A = 1$ identity

$= R^2(\cos^2\alpha + \sin^2\alpha) = R^2(1)$

$= 1^2 + 2^2$

$\therefore R^2 = 5 \qquad R = \cancel{\pm}\sqrt{5}$

given that $R > 0$

$\therefore R = \sqrt{5} \qquad \alpha = 1.107$

P 7 2 1 3 8 A 0 4 3 2

Question 2 continued

b) (i) $g(x) = 3 - 7f(2x)$

$f(x) = \sqrt{5}\cos(x - 1.107)$

$-\sqrt{5} \leqslant f(x) \leqslant \sqrt{5}$

$-\sqrt{5} \leqslant f(2x) \leqslant \sqrt{5}$

$3 + 7\sqrt{5} \geqslant g(x) \geqslant 3 - 7\sqrt{5}$

$\therefore$ max value of $g(x) = 3 + 7\sqrt{5}$

(ii) max value occurs when $f(2x) = -\sqrt{5}$

$\cancel{\sqrt{5}}\cos(2x - 1.107) = -\cancel{\sqrt{5}}$

$\cos(2x - 1.107) = -1$

$2x - 1.107 = \pi$

$x = 2.12$

(Total for Question 2 is 6 marks)

3.

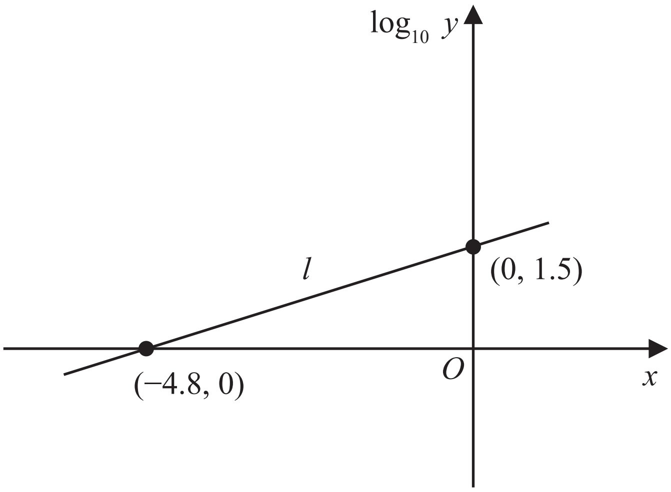


Figure 1

The line l in Figure 1 shows a linear relationship between $\log_{10} y$ and x.

The line passes through the points (0, 1.5) and (−4.8, 0) as shown.

(a) Write down an equation for l.

(2)

(b) Hence, or otherwise, express y in the form kb^x, giving the values of the constants k and b to 3 significant figures.

(3)

3.a) Equation of line : $y - y_1 = m(x - x_1)$ coordinates of known point on line; gradient

Known points : $(-4.8, 0)$ & $(0, 1.5)$

gradient $= \dfrac{\Delta y}{\Delta x} = \dfrac{1.5 - (0)}{0 - (-4.8)} = \dfrac{5}{16}$

$\log_{10} y - 0 = \dfrac{5}{16}(x - (-4.8))$

$l : \log_{10} y = \dfrac{5}{16}x + \dfrac{3}{2}$

Question 3 continued

b) $\log_{10} y = \frac{5}{16}x + \frac{3}{2}$

LOG RULES $\rightarrow$ $\log_a b = c$ $\rightarrow$ $a^c = b$

$y = 10^{\frac{5x}{16} + \frac{3}{2}}$

$y = \left(10^{\frac{5}{16}}\right)^x \times 10^{\frac{3}{2}}$

$y = 2.05^x \times 31.6$

(Total for Question 3 is 5 marks)

4. $$f(x) = \frac{2x^4 + 15x^3 + 35x^2 + 21x - 4}{(x+3)^2} \qquad x \in \mathbb{R} \quad x > -3$$

(a) Find the values of the constants A, B, C and D such that

$$f(x) = Ax^2 + Bx + C + \frac{D}{(x+3)^2}$$

(4)

(b) Hence find,

$$\int f(x)\,dx$$

(3)

4. a) $f(x) = \dfrac{2x^4 + 15x^3 + 35x^2 + 21x - 4}{(x+3)^2}$

$$\begin{array}{r|l} & 2x^2 + 3x - 1 \\ x^2+6x+9 & 2x^4 + 15x^3 + 35x^2 + 21x - 4 \\ - & 2x^4 + 12x^3 + 18x^2 \\ \hline & 3x^3 + 17x^2 + 21x \\ - & 3x^3 + 18x^2 + 27x \\ \hline & -x^2 - 6x - 4 \\ - & -x^2 - 6x - 9 \\ \hline & 5 \end{array}$$

$\therefore\ 2x^4 + 15x^3 + 35x^2 + 21x - 4 = (x+3)^2(2x^2+3x-1)+5$

$\therefore\ f(x) = 2x^2 + 3x - 1 + \dfrac{5}{(x+3)^2}$

$A = 2$
$B = 3$
$C = -1$
$D = 5$

b) $\int f(x)\,dx$

$= \int 2x^2 + 3x - 1 + \dfrac{5}{(x+3)^2}\,dx$

$= \left[\frac{2}{3}x^3 + \frac{3}{2}x^2 - x\right] + \int \dfrac{5}{(x+3)^2}\,dx$

P 7 2 1 3 8 A 0 8 3 2

Question 4 continued

$u = x+3 \qquad \frac{du}{dx} = 1 \rightarrow dx = du$

$\int \frac{5}{(x+3)^2}\, dx$

$= \int \frac{5}{u^2}\, du = \left[-5u^{-1}\right] = \left[-5(x+3)^{-1}\right]$

$\int f(x)\, dx = \frac{2}{3}x^3 + \frac{3}{2}x^2 - x - \frac{5}{x+3} + c$

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2\theta$$

giving your answers to 2 decimal places.

(5)

5. a) LHS : $\cot^2(x) - \tan^2(x)$ RHS : $4\cot(2x)\operatorname{cosec}(2x)$

$$= \frac{\cos^2(x)}{\sin^2(x)} - \frac{\sin^2(x)}{\cos^2(x)}$$

$$= \frac{\cos^4(x) - \sin^4(x)}{\sin^2(x)\cos^2(x)}$$

$$= \frac{(\cos^2(x)+\sin^2(x))(\cos^2(x)-\sin^2(x))}{\sin^2(x)\cos^2(x)}$$

$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$

$\boxed{\cos^2(A) + \sin^2(A) = 1}$

$\boxed{\cos(2A) = \cos^2(A) - \sin^2(A)}$

$$= \frac{(1)\cos(2x)}{\left(\frac{\sin(2x)}{2}\right)^2}$$

$$= \frac{4\cos(2x)}{\sin^2(2x)}$$

$$= \frac{4\cos(2x)}{\sin(2x)} \times \frac{1}{\sin(2x)} = 4\cot(2x)\operatorname{cosec}(2x) = \text{RHS}$$

Question 5 continued

b) $4\cot(2\theta)\operatorname{cosec}(2\theta) = 2\tan^2(\theta)$ $\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$\cot^2(\theta) - \tan^2(\theta) = 2\tan^2(\theta)$$

$$\cot^2(\theta) = 3\tan^2(\theta)$$

$$\frac{1}{\tan^2(\theta)} = 3\tan^2(\theta)$$

$$\tan^4(\theta) = \frac{1}{3}$$

$$\tan(\theta) = \pm\left(\frac{1}{3}\right)^{\frac{1}{4}}$$

$\theta = 0.650 \ \cup \ -0.650$

6.

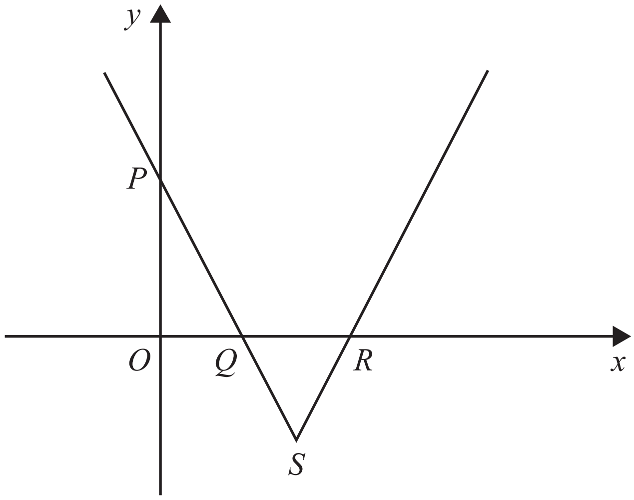


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = |3x - 5a| - 2a$$

where a is a positive constant.

The graph

- cuts the y-axis at the point P
- cuts the x-axis at the points Q and R
- has a minimum point at S

(a) Find, in simplest form in terms of a, the coordinates of

(i) point P

(ii) points Q and R

(iii) point S

(4)

(b) Find, in simplest form in terms of a, the values of x for which

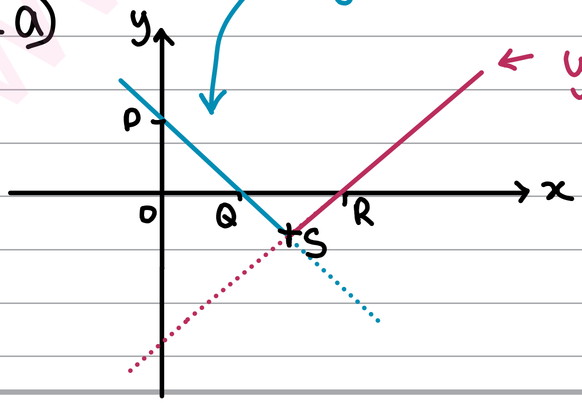
$$|3x - 5a| - 2a = |x - 2a|$$

(4)

$y = -(3x - 5a) - 2a = -3x + 3a$

6.a)

$y = (3x - 5a) - 2a = 3x - 7a$



Question 6 continued

(i) P is the y-intercept of $y = -3x + 3a$

$\therefore P = (0, 3a)$

(ii) Q is the x-intercept of $y = -3x + 3a$

$0 = -3x + 3a$

$x = a \rightarrow (a, 0)$

R is the x-intercept of $y = 3x - 7a$

$0 = 3x - 7a$

$x = \frac{7a}{3} \rightarrow \left(\frac{7a}{3}, 0\right)$

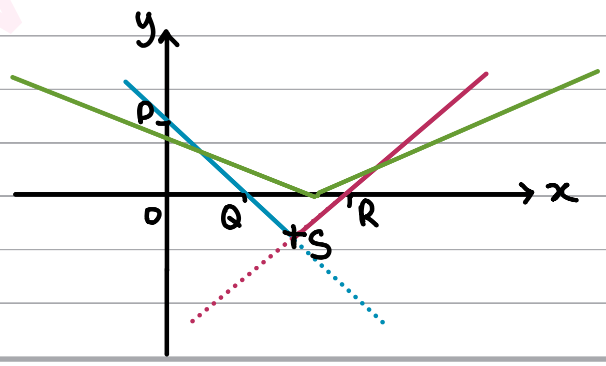
(iii) S is the intersection of both parts of the moduli

$-3x + 3a = 3x - 7a$

$6x = 10a$

$x = \frac{5a}{3} \rightarrow \left(\frac{5a}{3}, -2a\right)$

b) $|3x - 5a| - 2a = |x - 2a|$



We can see from the graph that the positive section of $|x-2a|$ intersects with $y = 3x - 7a$

& the negative section of $|x-2a|$ intersects with $y = -3x + 3a$

Question 6 continued

$\therefore \quad -3x+3a = -x+2a \qquad\qquad 3x-7a = x-2a$

$2x = a \qquad\qquad 2x = 5a$

$x = \frac{a}{2} \qquad\qquad x = \frac{5a}{2}$

P 7 2 1 3 8 A 0 1 8 3 2

7. The curve C has equation

$$x = 3\tan\left(y - \frac{\pi}{6}\right) \qquad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are integers to be found.

(4)

The point P with y coordinate $\frac{\pi}{3}$ lies on C.

Given that the tangent to C at P crosses the x-axis at the point Q.

(b) find, in simplest form, the exact x coordinate of Q.

(5)

7. a) $x = 3\tan\left(y - \frac{\pi}{6}\right)$ $\quad x \in R, \; -\frac{\pi}{3} < y < \frac{2\pi}{3}$

$$\frac{dx}{dy} = 3\sec^2\left(y - \frac{\pi}{6}\right)$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\left(dx/dy\right)}}$$

$\sin^2 A + \cos^2 A = 1$ ← divide through by $\cos^2 A$

$$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$$

$$\boxed{\tan^2 A + 1 = \sec^2 A}$$

$$\frac{dy}{dx} = \frac{1}{3\sec^2\left(y - \frac{\pi}{6}\right)} = \frac{1}{3} \times \frac{1}{\tan^2\left(y - \frac{\pi}{6}\right) + 1}$$

$$= \frac{1}{3\tan^2\left(y - \frac{\pi}{6}\right) + 3}$$

$$= \frac{1}{\frac{x^2}{3} + 3} = \frac{3}{x^2 + 9}$$

DO NOT WRITE IN THIS AREA

Question 7 continued

b) grad. tangent to C $= \frac{dy}{dx}$

$C \rightarrow \left(\sqrt{3}, \frac{\pi}{3}\right)$

$\frac{dy}{dx}(C) = \frac{3}{(\sqrt{3})^2 + 9} = \frac{1}{4}$ ← gradient of tangent

Equation of line : $y - y_1 = m(x - x_1)$ coordinates of known point on line

$\uparrow$ gradient

$y - \frac{\pi}{3} = \frac{1}{4}(x - \sqrt{3})$

If the tangent crosses the x-axis at Q → $(q, 0)$

$0 - \frac{\pi}{3} = \frac{1}{4}(q - \sqrt{3})$

$q = 4\left(\frac{\sqrt{3}}{4} - \frac{\pi}{3}\right) = \sqrt{3} - \frac{4\pi}{3}$

so $x = \sqrt{3} - \frac{4\pi}{3}$

8. Find, in simplest form,

$$\int (2\cos x - \sin x)^2 \, dx$$

(5)

8. $\int (2\cos(x) - \sin(x))^2 dx$

$= \int 4\cos^2(x) - 4\cos(x)\sin(x) + \sin^2(x) \, dx$

$\boxed{\sin(2A) = 2\sin(A)\cos(A)}$

$= \int -2\sin(2x) + 4\cos^2(x) + 1 - \cos^2(x) \, dx$

$= \int -2\sin(2x) + 1 + 3\cos^2(x) \, dx$

USING DOUBLE ANGLE FORMULAE $\rightarrow$ $\cos(2A) = \cos^2(A) - \sin^2(A)$

$= \cos^2(A) - (1 - \cos^2(A))$

$= 2\cos^2(A) - 1$

$\therefore \cos^2(2A) = \frac{1}{2}(\cos(2A) + 1)$

$= \int -2\sin(2x) + 1 + \frac{3}{2}(\cos(2x) + 1) \, dx$

$= \int -2\sin(2x) + \frac{3\cos(2x)}{2} + \frac{5}{2} \, dx$

$= \cos(2x) + \frac{3\sin(2x)}{4} + \frac{5x}{2} + c$

9.

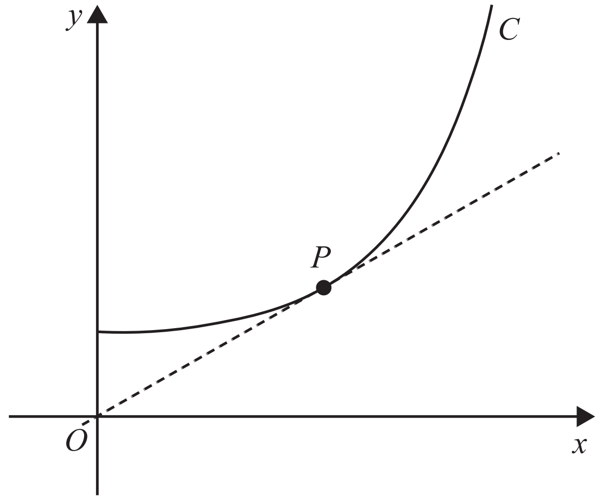


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \sqrt{3 + 4e^{x^2}} \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.
(2)

The point P with x coordinate α lies on C.

Given that the tangent to C at P passes through the origin, as shown in Figure 3,

(b) show that $x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(3)

(c) Hence show that α lies between 1 and 2
(2)

(d) Show that the equation in part (b) can be written in the form

$$x = \frac{1}{2}\sqrt{4 + 3e^{-x^2}}$$

(1)

The iteration formula

$$x_{n+1} = \frac{1}{2}\sqrt{4 + 3e^{-x_n^2}}$$

with $x_1 = 1$ is used to find an approximation for α.

(e) Use the iteration formula to find, to 4 decimal places, the value of

(i) x_3

(ii) α

(3)

Question 9 continued

9. a) $y = (3+4e^{x^2})^{\frac{1}{2}}$

$\boxed{\text{CHAIN RULE : } \frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}}$

$u = 3+4e^{x^2}$ $\qquad \frac{du}{dx} = (2x)(4e^{x^2}) = 8xe^{x^2}$

$y = u^{\frac{1}{2}}$ $\qquad \frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{2}u^{-\frac{1}{2}} \times 8xe^{x^2}$$

$$= 4xe^{x^2}(3+4e^{x^2})^{-\frac{1}{2}}$$

b) Tangent to P going through origin

↳ $y = mx$

$\uparrow \frac{dy}{dx}$

$P = (\alpha, (3+4e^{\alpha^2})^{\frac{1}{2}})$

substitute with given equation $\longrightarrow$ $y = \frac{4xe^{x^2}}{\sqrt{3+4e^{x^2}}}x$

$$\sqrt{3+4e^{x^2}} = \frac{4xe^{x^2}}{\sqrt{3+4e^{x^2}}}x$$

$$3+4e^{x^2} = 4x^2e^{x^2}$$

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

c) Let $f(x) = 4x^2e^{x^2} - 4e^{x^2} - 3$

$f(1) = -3 \qquad f(2) = 652...$

given that $f(x)$ is continuous between 1 & 2,

because there's a sign change → root lies between 1 & 2

d) $4x^2e^{x^2} - 4e^{x^2} - 3 = 0$

$x^2 = \dfrac{3 + 4e^{x^2}}{4e^{x^2}} = \dfrac{3}{4}e^{-x^2} + 1$

$x = \frac{1}{2}\sqrt{3e^{-x^2} + 4}$

$x_{n+1} = \frac{1}{2}\sqrt{4 + 3e^{-x_n^2}}$

$x_1 = 1$

$x_2 = x_{1+1} = \frac{1}{2}\sqrt{4 + 3e^{-(1)^2}} = 1.1296$

(i) $x_3 = 1.0997$

$x_4 = 1.1062$

$x_5 = 1.1048$

$x_6 = 1.1051$

(ii) $x_7 = 1.1051$ consistent to 4 d.p.

$\alpha = 1.1051$

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A population of fruit flies is being studied.

The number of fruit flies, F, in the population, t days after the start of the study, is modelled by the equation

$$F = \frac{350e^{kt}}{9 + e^{kt}}$$

where k is a constant.

Use the equation of the model to answer parts (a), (b) and (c).

(a) Find the number of fruit flies in the population at the start of the study. **(1)**

Given that there are 200 fruit flies in the population 15 days after the start of the study,

(b) show that $k = \frac{1}{15}\ln 12$ **(3)**

Given also that, when $t = T$, the number of fruit flies in the population is increasing at a rate of 10 per day,

(c) find the possible values of T, giving your answers to one decimal place. **(5)**

10. a) when $t = 0$ $\quad F_0 = \frac{350e^{k(0)}}{9 + e^{k(0)}} = \frac{350}{10} = 35$

b) when $t = 15$ $\quad F_{15} = \frac{350e^{k(15)}}{9 + e^{k(15)}} = 200$

$7e^{15k} = 36 + 4e^{15k}$

$3e^{15k} = 36$

$e^{15k} = 12$

$k = \frac{1}{15}\ln(12)$

Question 10 continued

c) $\frac{dF}{dt}$ = rate of increase of population

$$F = \frac{350e^{kt}}{9+e^{kt}}$$

Quotient rule for differentiating : $y = \frac{u}{v} \rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = 350e^{kt}$ $\qquad \frac{du}{dt} = 350ke^{kt}$

$v = 9+e^{kt}$ $\qquad \frac{dv}{dt} = ke^{kt}$

$$\frac{dF}{dt} = \frac{(9+e^{kt})(350ke^{kt}) - (350e^{kt})(ke^{kt})}{(9+e^{kt})^2}$$

$$= \frac{350e^{kt}(9k + \cancel{ke^{kt}} - \cancel{ke^{kt}})}{(9+e^{kt})^2}$$

when $t = T \rightarrow \frac{315\cancel{0}\,k\,e^{kT}}{(9+e^{kT})^2} = 1\cancel{0}$

$$315ke^{kT} = (9+e^{kT})^2$$

$$315ke^{kT} = 81 + 18e^{kT} + e^{2kT}$$

$$e^{2kT} + (18-315k)e^{kT} + 81 = 0$$

Question 10 continued

Quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore e^{kT} = \frac{-(18-315k) \pm \sqrt{(18-315k)^2 - 4(81)}}{2}$$

$$k = \frac{\ln(12)}{15}$$

$$\therefore e^{kT} = 31.62 \cup 2.562$$

$$T = \frac{1}{k}\ln(31.62) \cup \frac{1}{k}\ln(2.562)$$

$$= 20.8 \quad \cup \quad 5.7$$

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS